Zhengzhou Commodity Exchange Option Seminar

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Zhengzhou Commodity Exchange Option Seminar

Financial Contracts
Spot or Cash Transaction

An agreement between a buyer and seller whereby the price is negotiated right now, and is immediately followed by an exchange of money for goods.

Forward Contract

An agreement between a buyer and seller whereby the price is negotiated right now, but the actual exchange of money for goods takes place at a later date.
**Futures Contract**
an exchange-traded forward contract

**Option Contract**
the right to decide at a later date

**Call Option**
the right to decide whether to buy

**Put Option**
the right to decide whether to sell
Every option contract must specify:

- the goods to be bought or sold (the *underlying*)
- the date on which the buyer of the option must make the final decision (the *expiration date* or *expiry*)
- the amount to be paid for the goods (the *strike price* or the *exercise price*)

The premium or price to be paid for the option is negotiated between the buyer and seller of the option.
Stock-type Settlement

Buy 100 shares of stock at $65 per share

\[
\text{total cost} = 100 \times 65 = 6,500
\]

Stock price rises to $75

\[
\text{profit} = 100 \times +10 = +1,000 \quad \text{(unrealized)}
\]

Sell stock at $75 per share

\[
\text{profit} = 100 \times +10 = +1,000 \quad \text{(realized)}
\]
Futures-type Settlement

The *nominal value* or *notional value* of a commodity futures contract:

the unit value multiplied by the number of units to be delivered

\[
\text{unit value} = 1400 \\
\text{units to be delivered} = 100 \\
\text{nominal value} = 1400 \times 100 \\
= 140,000
\]
Futures-type Settlement

Buy a gold futures contract at 1400

\[
\text{total value of trade} = 1400 \times 100 = 140,000
\]

payment from buyer to seller = 0

*margin* deposit with clearing house

Futures contract rises to 1420

\[
\text{profit} = 20 \times +100 = +2,000 \quad (\text{realized})
\]

Futures contract falls to 1380

\[
\text{loss} = -20 \times 100 = -2,000 \quad (\text{realized})
\]

*Variation* – the daily realized profit or loss on an open futures position
Settlement of Exchange-Traded Contracts:

Stock is always subject to stock-type settlement

Futures are always subject to futures-type settlement (sometimes referred to as *margin and variation*)

Options may be subject to either stock-type or futures-type settlement.
On most option markets around the world options are subject to the same settlement procedure as the underlying contract –

If the underlying for the option is stock (or a security) the options are subject to stock-type settlement

If the underlying for the option is a futures contract the options are subject to futures-type settlement

Settlement of Exchange-Traded Contracts:
Settlement of Exchange-Traded Contracts:

In the United States all options, whether options on stock or options on futures, are subject to stock-type settlement.

All options must be paid for fully in cash.
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Option Contract Specifications
**Underlying Contract** — the security or commodity to be bought or sold

- stock
- stock index
- futures contract
- physical commodity
- interest rate product
Expiration Date (Expiry) – the date on which the option buyer must make a final decision whether to buy or sell

Exercise Price or Strike Price – the price at which the underlying contract will be bought or sold

Type of contract – either a call (the right to buy) or a put (the right to sell)
Sugar
underlying contract
Cotton
September
expiration date
March
5000
exercise price
20000
Call
type
Put
Premium – the price paid for an option

Options may be subject to either...

stock-type settlement

futures-type settlement
**Exercise** – the process by which the buyer of an option converts the option into a long position in the underlying contract (a call) or a short position in the underlying contract (a put).

**Assignment** – the process by which the seller of an option is required to take a short position in the underlying contract (a call) or a long position in the underlying contract (a put).
If you ...

exercise a call, you choose to buy at the exercise price
are assigned on a call, you are forced to sell at the exercise price

exercise a put, you choose to sell at the exercise price
are assigned on a put, you are forced to buy at the exercise price
Exercise style

**European** – the option may be exercised only at expiration. Most stock index options are European.

**American** – the option may be exercised at any time prior to expiration. Most individual stock options and futures options are American.
Serial Option Months — futures options with expiration months which do not match a futures contract

When exercised, a serial option results in a position in the nearest futures contract beyond the option expiration.

With trading in March, June, September, and December futures contracts……

An April or May option will exercise into a June future
A July or August option will exercise into a September future
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Intrinsic Value and Time Value
An option’s price is made up of its

*intrinsic value*

*time value* (or *time premium*).

An option has intrinsic value if it enables the holder to buy low and sell high, or sell high and buy low.

The intrinsic value is the difference between the buying and selling price.
You own a November 100 call with the underlying contract trading at 115. If you exercise the call you can:

buy at 100 / sell at 115  (buy low / sell high)

intrinsic value = 15

You own a March 150 put with the underlying contract trading at 130. If you exercise the put you can:

sell at 150 / buy at 130  (sell high / buy low)

intrinsic value = 20
An option’s intrinsic value can never be less than zero since no one would choose to buy high and sell low, or sell low and buy high. Therefore...

\[
\text{call intrinsic value} = \text{maximum } [0, S-X]
\]

\[
\text{put intrinsic value} = \text{maximum } [0, X-S]
\]
Since

\[ \text{price} = \text{intrinsic value} + \text{time value} \]

\[ \text{time value} = \text{price} - \text{intrinsic value} \]

An option’s intrinsic value is sometimes referred to as \textit{parity}. 
Futures trading at 1348

December 1300 call trading at 73

\[
\text{price} = \text{intrinsic value} + \text{time value}
\]

\[
73 = 48 + 25
\]

\[
(1348 - 1300) + (73 - 48)
\]
Futures trading at 2369

November 2400 put trading at 51

\[
\text{price} = \text{intrinsic value} + \text{time value}
\]

\[
51 = 31 + 20
\]

\[
(2400 - 2369) + (51 - 31)
\]
Depending on an option’s exercise price and the price of the underlying contract, an option is said to be either.....

- *in-the-money*

- *at-the-money*

- *out-of-the-money*
in-the-money call: \( S > X \)

in-the-money put: \( X > S \)

(intrinsic value greater than zero)

out-of-the-money call: \( S \leq X \)

out-of-the-money put: \( X \leq S \)

(no intrinsic value)

at-the-money call or put: \( S = X \)
current underlying price

in-the-money calls
lower exercise prices
out-of-the-money puts

out-of-the-money calls
higher exercise prices
in-the-money puts

at-the-money calls and puts
underlying futures contract = 8510

<table>
<thead>
<tr>
<th>exercise price</th>
<th>call</th>
<th>put</th>
</tr>
</thead>
<tbody>
<tr>
<td>8000</td>
<td>I-T-M by 510</td>
<td>O-T-M by 510</td>
</tr>
<tr>
<td>8500</td>
<td>I-T-M by 10</td>
<td>O-T-M by 10</td>
</tr>
<tr>
<td>9000</td>
<td>O-T-M by 490</td>
<td>I-T-M by 490</td>
</tr>
</tbody>
</table>

The 8500 call and put are the at-the-money options.
Expiration Profit and Loss Graphs
At expiration, an option is worth exactly its intrinsic value (parity)

- zero if the option is out-of-the-money
- the difference between the exercise price and underlying price if the option is in-the-money

*Parity Graph*  – a graph which represents the value of an option, or option position, at expiration
Long Call

Lower prices  
exercise price  
higher prices
Short Call

lower prices    higher prices

exercise price

Short Call

0

+1

-1
Long Put

(lower prices) → (higher prices)

exercise price

+1

-1

0
Short Put

lower prices  higher prices

exercise price

Short Put
Slope = \frac{\text{change in option value}}{\text{change in underlying price}}

<table>
<thead>
<tr>
<th></th>
<th>out-of-the-money</th>
<th>in-the-money</th>
</tr>
</thead>
<tbody>
<tr>
<td>long call</td>
<td>0</td>
<td>+1</td>
</tr>
<tr>
<td>short call</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>long put</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>short put</td>
<td>0</td>
<td>+1</td>
</tr>
</tbody>
</table>
long call and long put at the same exercise price

slope = -1

below exercise price
slope = 0 - 1 = -1

slope = 0

above exercise price
price = 0 + 1 = +1

slope = +1

exercise price
current price

slope = +1

long underlying
current price

slope = -1

short underlying
long call and short put, same exercise price

below exercise price
slope = 0 + 1 = +1

slope = 0

above exercise price
slope = 0 + 1 = +1
The profit and loss graph for an option position at expiration is the parity graph …

shifted downward by the amount paid when an option is purchased

or

shifted upward by the amount received when an option is sold
buy a 100 call at a price of 4.00

break-even
sell a 95 call at a price of 6.50
buy a 105 put at a price of 5.25

break-even
sell a 95 put at a price of 1.75

93.25

+1.75

break-even
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Theoretical Pricing Models
What is the *theoretical value* of a roulette bet?

1, 2, 3, …… , 34, 35, 36, 0, 00
Choose one of 38 numbers

If your number doesn’t come up you receive nothing

If your number does come up you receive $36

*Expected value* (expected return)

\[
\frac{36}{38} \approx 95\text{¢}
\]
Expected value

• depends primarily on the laws of probability

• does not have to correspond to a possible outcome

• is only reliable in the long run
The price for the privilege of choosing a number is $1.00

The casino has an *edge* of

\[ $1.00 - 95\text{¢} = 5\text{¢} \]

The price for the privilege of choosing a number is 88¢

The player has an edge of

\[ 95\text{¢} - 88\text{¢} = 7\text{¢} \]
Theoretical value (theoretical price, fair value, fair price):

The price you would be willing to pay now in order to just break even in the long run.

- expected value

- other considerations
If your number doesn’t come up you receive receive nothing.

If your number does come up you receive $36, to be paid in two months.

interest rates $= 12.00\%$

theoretical value

$= \text{present value of 95¢}$

$= 95¢ / (1 + 2/12\times12\%)$

$= 95¢ / 1.02 \approx 93¢$
Theoretical edge

The difference between the price of a proposition and its theoretical value

\[ \$1.00 - 93\¢ = 7\¢ \]

Positive theoretical edge

- buy at a price lower than theoretical value
  - or
  - sell at a price higher than theoretical value
Intelligent trading of options requires us to ….

• calculate a theoretical value

• choose an appropriate strategy

• control the risk
A model is a representation of the real world

A model is unlikely to be an exact representation of the real world

A model is limited by

- the accuracy of the assumptions on which the model is based
- the accuracy of the inputs into the model
Theoretical option pricing model

A mathematical model used to determine the theoretical value of an option contract under some set of assumptions about

- market conditions
- the terms of the option contract
Expected value for the underlying?

\[ 20\% \times 90 + 20\% \times 95 \ldots + 20\% \times 110 = 100 \]
Present value? 2 months 12% per year

$\frac{3.00}{1+12\%\times\frac{2}{12}} \approx 2.94$

Expected value for the 100 call?

$20\%\times0 + 20\%\times0 + 20\%\times0 + 20\%\times5 + 20\%\times10$

$= 3.00$
Should every outcome be equally likely?

Expected value for the underlying? 100

Expected value for 100 call?

\[20\% \times 5 + 10\% \times 10 = 2.00\]
What should be the most likely underlying price at expiration? forward price
Must the probabilities be symmetrical?

Expected value for the underlying?

\[5\% \times 90 + 20\% \times 95 + 40\% \times 100 + 25\% \times 105 + 10\% \times 110 = 100.75\]

100 call?

\[25\% \times 5 + 10\% \times 10 = 2.25\]
All theoretical pricing models attempt to:

• propose a series of prices for the underlying contract at expiration

• assign appropriate probabilities to each underlying price, with the restriction that the expected value of the underlying contract must be equal to the forward price

• using the exercise price, underlying prices, and probabilities, calculate the option’s expected value at expiration

• calculate the option’s present value (theoretical value) by discounting the expected value by the appropriate interest component
Theoretical Pricing Models

- exercise price
- time to expiration
- underlying price
- interest rate
- volatility

pricing model → theoretical value
Expected Value Exercise

Using the above prices and probabilities for an underlying contract, what are the expected values for the following contracts:

- Underlying
- 126 call
- 130 call
- 133 call
- 126 put
- 130 put
- 133 put

What do you notice about the difference between the values of calls and puts at the same exercise price?
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Volatility
What is volatility?
probabilities

underlying prices
underlying prices
normal distribution

underlying prices
All normal distributions are defined by their mean and their standard deviation.
mean

half of the distribution is to the left of the mean

half of the distribution is to the right of the mean
low standard deviation
high peak
narrow body

high standard deviation
low peak
wide body
±.25 each day
value = .05

±1.50 each day
value = .75

±10.00 each day
value = 8.00

90 days to expiration

120 call

option value
90 days to expiration

80 put

120 call

Option value
+1 S.D. ≈ 34%  
-1 S.D. ≈ 34%  
+2 S.D. ≈ 47.5%  
-2 S.D. ≈ 47.5%  
± 1 S.D. ≈ 68% (2/3)  
± 2 S.D. ≈ 95% (19/20)
We would expect to see an occurrence

- within 1 standard deviation
  approx. 2 times out of 3

- within 2 standard deviations
  approx. 19 times out of 20

- greater than 1 standard deviation
  approx. 1 time in 3

- greater than 2 standard deviations
  approx. 1 time in 20
exercise price

time to expiration

underlying price

interest rate

volatility

(mean?)

standard deviation?

(dividends)
Mean – forward price
(underlying price, time to expiration, interest rates, dividends)

Standard deviation – volatility

Volatility: one standard deviation, in percent, over a one year period.
1-year forward price = 100.00
volatility = 20%

One year from now:

• 2/3 chance the contract will be between 80 and 120 (100 ± 20%)

• 19/20 chance the contract will be between 60 to 140 (100 ± 2*20%)

• 1/20 chance the contract will be less than 60 or more than 140
What does an annual volatility tell us about movement over some other time period?

monthly price movement?
weekly price movement?
daily price movement?

Volatility$_t$ = Volatility$_{annual}$ * $\sqrt{t}$
Daily volatility (standard deviation)

Trading days in a year? 250 – 260

Assume 256 trading days

\[ t = \frac{1}{256} \quad \sqrt{t} = \sqrt{\frac{1}{256}} = \frac{1}{16} \]

\[ \text{Volatility}_{\text{daily}} = \frac{\text{Volatility}_{\text{annual}}}{16} \]
current price = 100.00

volatility_{daily} \approx \frac{20\%}{16} = 1\frac{1}{4}\%

One trading day from now:

- 2/3 chance the contract will be between 98.75 and 101.25
  
  \[(100 \pm 1\frac{1}{4}\%)
  
- 19/20 chance the contract will be between 97.50 and 102.50
  
  \[(100 \pm 2\times1\frac{1}{4}\%)

84
Weekly volatility:

\[ t = \frac{1}{52} \quad \sqrt{t} = \sqrt{\frac{1}{52}} \approx \frac{1}{7.2} \]

\[ \text{Volatility}_{\text{weekly}} \approx \frac{\text{Volatility}_{\text{annual}}}{7.2} \]

Monthly volatility:

\[ t = \frac{1}{12} \quad \sqrt{t} = \sqrt{\frac{1}{12}} \approx \frac{1}{3.5} \]

\[ \text{Volatility}_{\text{monthly}} \approx \frac{\text{Volatility}_{\text{annual}}}{3.5} \]
Volatility Exercise I

For each contract and volatility below, what would be an approximate daily and weekly standard deviation:

<table>
<thead>
<tr>
<th>Sugar futures trading at 5140</th>
<th>10%</th>
<th>12%</th>
<th>14%</th>
<th>16%</th>
</tr>
</thead>
<tbody>
<tr>
<td>daily</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>weekly</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Glass futures trading at 1465</th>
<th>15%</th>
<th>20%</th>
<th>25%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>daily</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>weekly</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Volatility Exercise II

For each contract, volatility, and time interval below, what would be an approximate one standard deviation price change:

Cotton futures trading at 19,600

- volatility = 7.5%, time = 22 days
- volatility = 11.25%, time = 86 days

Wheat futures trading at 2625.00

- volatility = 14%, time = 9 weeks
- volatility = 9.75%, time = 27 weeks
futures price = 2628; volatility = 14%

daily standard deviation

\[ \approx \frac{2628 \times 14}{16} \]
\[ = 2628 \times 0.875 \%
\approx 23 \]

weekly standard deviation

\[ \approx \frac{2628 \times 14}{7.2} \]
\[ = 2628 \times 1.94 \%
\approx 51 \]
futures price = 2628; volatility = 14%

daily standard deviation = 23

+8  +19  -12  -21  +16

Is 14% a reasonable volatility estimate?

How often do you expect to see an occurrence greater than one standard deviation?
normal distribution

lognormal distribution
forward price = 100

<table>
<thead>
<tr>
<th></th>
<th>normal distribution</th>
<th>lognormal distribution</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>110 call</td>
<td>3.00</td>
<td>3.20</td>
<td>2.90</td>
</tr>
<tr>
<td>90 put</td>
<td>3.00</td>
<td>2.80</td>
<td>3.10</td>
</tr>
</tbody>
</table>

Are the options mispriced?

Maybe the marketplace thinks the model is wrong.
Maybe the marketplace is right.
**realized volatility:** The volatility of the underlying contract over some period of time (historical, future)

**implied volatility:**

- derived from the prices of options in the marketplace
- the marketplace’s consensus forecast of future volatility
Implied volatility

31%

74

27%

68

Spot price
Strike price
Time
Interest rate
Volatility

Pricing model
Theoretical value
today

realized volatility

backward looking

(what has occurred)

implied volatility

forward looking

(what the marketplace thinks will occur)

implied volatility = price

realized volatility = value

(historical, future)
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Risk Measurement
Trade an underlying futures contract:

- risk

Trade an option:

- directional
- volatility
- time
- interest rate
**Delta (Δ)** –

The number of underlying contracts required to establish a neutral hedge

The directional risk of a position in terms of an equivalent position in the underlying contract

The sensitivity of an option’s theoretical value to a change in the price of the underlying contract

Calls have positive deltas / Puts have negative deltas

\[ \Delta = 50 \times 0.50 \]

- **underlying price**: up 1.20, down 1.70
- **option value**: up 0.60, down 0.85
calls have positive delta values
puts have negative delta values

If….

underlying price \uparrow \quad \text{call values} \uparrow

underlying price \downarrow \quad \text{put values} \downarrow

\text{call values} \downarrow \quad \text{put values} \uparrow
Delta: approximately the probability that an option will finish in-the-money

10 (-10)  10% chance of finishing in-the-money
90 (-90)  90% chance of finishing in-the-money
50 (-50)  50% chance of finishing in-the-money

less than 50 (-50): out-of-the-money

greater than 50 (-50): in-the-money

equal to 50 (-50): at-the-money
Option delta = 10

Probability that the option will finish in-the-money: 10%

Probability that the option will finish out-of-the-money: 90%

Sell option at 1.00

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>out-of-the-money</td>
<td>9 * +1.00</td>
</tr>
<tr>
<td>in-the-money</td>
<td>1 * -20.00</td>
</tr>
</tbody>
</table>

total expected P&L -11.00
**Gamma or curvature** ($\Gamma$) –

The rate of change in an option’s delta with respect to movement in the price of the underlying contract

Usually expressed as the change in the delta per one point change in the price of the underlying contract

All options have positive gamma values
Underlying price = 100

<table>
<thead>
<tr>
<th>Underlying Price</th>
<th>Delta</th>
<th>Gamma</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>101</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>102</td>
<td>34</td>
<td></td>
</tr>
</tbody>
</table>

Underlying price falls to 75

New delta of 110 call?

\[ 30 - 25 \times 2 = -10 \]

The gamma must be changing
All options have positive gamma values

underlying price = 100 → 101

→ 99

100 call
delta = +50
gamma = +5

+50 +50
+5 -5
+55 +45

100 put
delta = -50
gamma = +5

-50 -50
+5 -5
-45 -55
Delta = speed
rate of change in the option value
(first derivative)

Gamma = acceleration
rate of change in the delta value
(second derivative)
**Theta (Θ):**

The sensitivity of an option’s value to the passage of time

Usually expressed as the change in value per one day’s passage of time

Option value = 5.00 \hspace{1cm} \text{theta} = .05

+1 day \hspace{1cm} 4.95
+2 days \hspace{1cm} 4.90
+3 days \hspace{1cm} 4.85

Often written with a negative sign to represent a loss in value as time passes. Using this notation all options have negative theta values.
**Vega or Kappa (K) –**

The sensitivity of an option’s theoretical value to a change in volatility

Usually expressed as the change in value per one percentage point change in volatility

Often interpreted as the change in price with respect to a change in implied volatility

All options have positive vega values
Volatility = 25%

Option value = 5.00  
Vega = .20

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Option Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>25%</td>
<td>5.00</td>
</tr>
<tr>
<td>26%</td>
<td>5.20</td>
</tr>
<tr>
<td>27%</td>
<td>5.40</td>
</tr>
<tr>
<td>28%</td>
<td>5.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Vega</th>
</tr>
</thead>
<tbody>
<tr>
<td>24%</td>
<td>4.80</td>
</tr>
<tr>
<td>23%</td>
<td>4.60</td>
</tr>
<tr>
<td>22%</td>
<td>4.40</td>
</tr>
</tbody>
</table>

All options have positive vega values: they become more valuable as volatility rises and less valuable as volatility falls.
**Rho (P)** –

The sensitivity of an option’s theoretical value to a change in interest rates

Usually expressed as the change in value per one percentage point change in interest rates

The rho of an option may be either positive, negative, or zero depending on the type of option, the underlying contract, and the settlement procedure.
<table>
<thead>
<tr>
<th>measure (futures)</th>
<th>calls</th>
<th>puts</th>
<th>underlying</th>
</tr>
</thead>
<tbody>
<tr>
<td>delta</td>
<td>positive</td>
<td>negative</td>
<td>positive</td>
</tr>
<tr>
<td>gamma</td>
<td>positive</td>
<td>positive</td>
<td>zero</td>
</tr>
<tr>
<td>theta</td>
<td>negative</td>
<td>negative</td>
<td>zero</td>
</tr>
<tr>
<td>vega</td>
<td>positive</td>
<td>positive</td>
<td>zero</td>
</tr>
<tr>
<td>rho</td>
<td>zero</td>
<td>zero</td>
<td>zero</td>
</tr>
</tbody>
</table>
Risk Measurement Exercise

For each option on the following page:

1. If we assume that the delta is constant what will be the new theoretical value if the underlying contract moves by the given amount?

2. What will be the new delta if the underlying contract moves by the given amount?

3. If the underlying contract moves by the given amount what will be the approximate theoretical value if you also include the gamma? (Hint: What is the average delta?)

4. What will be the approximate theoretical value if ten days pass with no movement in the underlying contract?

5. What will be the approximate theoretical value if volatility changes by the given amount?
## Risk Measurement Exercise

<table>
<thead>
<tr>
<th>theoretical value</th>
<th>delta</th>
<th>gamma</th>
<th>daily theta</th>
<th>vega</th>
<th>underlying movement</th>
<th>change in volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 8.04</td>
<td>65</td>
<td>3.7</td>
<td>-.036</td>
<td>.24</td>
<td>↑ 3.00</td>
<td>↓ 3%</td>
</tr>
<tr>
<td>b) 1.88</td>
<td>-28</td>
<td>2.3</td>
<td>-.021</td>
<td>.30</td>
<td>↑ 2.50</td>
<td>↑ 7%</td>
</tr>
<tr>
<td>c) 3.76</td>
<td>50</td>
<td>4.9</td>
<td>-.012</td>
<td>.80</td>
<td>↓ 1.44</td>
<td>↑ 3.5%</td>
</tr>
<tr>
<td>d) 17.12</td>
<td>-87</td>
<td>2.9</td>
<td>-.060</td>
<td>.75</td>
<td>↓ 2.68</td>
<td>↓ 9%</td>
</tr>
<tr>
<td>e) .95</td>
<td>11</td>
<td>1.9</td>
<td>-.002</td>
<td>.06</td>
<td>↑ .66</td>
<td>↓ 2.5%</td>
</tr>
<tr>
<td>f) 14.56</td>
<td>-44</td>
<td>8.8</td>
<td>-.045</td>
<td>.92</td>
<td>↓ 10.00</td>
<td>↑ 6%</td>
</tr>
</tbody>
</table>
## Risk Measurement Exercise

<table>
<thead>
<tr>
<th></th>
<th>new theoretical value using a constant delta</th>
<th>new theoretical value using the average delta</th>
<th>if ten days pass</th>
<th>if volatility changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>original delta</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a)</td>
<td>65</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td>-28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d)</td>
<td>-87</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e)</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f)</td>
<td>-44</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Futures Price = 99.75  
Time to November Expiration = 3 months  
Volatility = 20.00%

<table>
<thead>
<tr>
<th>Exercise Price</th>
<th>Call Theoretical Price</th>
<th>Call Theoretical Value</th>
<th>Call Delta</th>
<th>Call Gamma</th>
<th>Call Theta</th>
<th>Call Vega</th>
<th>Put Theoretical Price</th>
<th>Put Theoretical Value</th>
<th>Put Delta</th>
<th>Put Gamma</th>
<th>Put Theta</th>
<th>Put Vega</th>
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<tbody>
<tr>
<td>90</td>
<td>10.74</td>
<td>10.49</td>
<td>86</td>
<td>2.2</td>
<td>-.0122</td>
<td>.111</td>
<td>.97</td>
<td>.74</td>
<td>-14</td>
<td>2.2</td>
<td>-.0122</td>
<td>.111</td>
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<tr>
<td>95</td>
<td>6.90</td>
<td>6.71</td>
<td>70</td>
<td>3.5</td>
<td>-.0189</td>
<td>.172</td>
<td>2.18</td>
<td>1.96</td>
<td>-30</td>
<td>3.5</td>
<td>-.0189</td>
<td>.172</td>
</tr>
<tr>
<td>100</td>
<td>3.94</td>
<td>3.85</td>
<td>51</td>
<td>4.0</td>
<td>-.0218</td>
<td>.199</td>
<td>4.17</td>
<td>4.10</td>
<td>-49</td>
<td>4.0</td>
<td>-.0218</td>
<td>.199</td>
</tr>
<tr>
<td>105</td>
<td>1.92</td>
<td>1.98</td>
<td>32</td>
<td>3.6</td>
<td>-.0196</td>
<td>.178</td>
<td>7.16</td>
<td>7.23</td>
<td>-68</td>
<td>3.6</td>
<td>-.0196</td>
<td>.178</td>
</tr>
<tr>
<td>110</td>
<td>.84</td>
<td>.91</td>
<td>18</td>
<td>2.6</td>
<td>-.0142</td>
<td>.129</td>
<td>11.11</td>
<td>11.16</td>
<td>-82</td>
<td>2.6</td>
<td>-.0142</td>
<td>.129</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Position</th>
<th>Theoretical Edge</th>
<th>Delta Position</th>
<th>Gamma Position</th>
<th>Theta Position</th>
<th>Vega Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long 7 futures contracts</td>
<td>0</td>
<td>+7 x +100</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Short 10 November 95 calls</td>
<td>10 x +.19</td>
<td>-10 x +70</td>
<td>-10 x +3.5</td>
<td>-10 x -.0196</td>
<td>-10 x +.172</td>
</tr>
<tr>
<td>Short 10 November 100 calls</td>
<td>20 x +.06</td>
<td>+20 x +32</td>
<td>+20 x +3.6</td>
<td>+20 x -.0196</td>
<td>+20 x +.178</td>
</tr>
<tr>
<td>Long 20 November 105 calls</td>
<td>10 x +.09</td>
<td>-10 x +51</td>
<td>+2.10</td>
<td>+2.10</td>
<td>+1.570</td>
</tr>
<tr>
<td>Long 10 November 110 calls</td>
<td>10 x +.07</td>
<td>+10 x +18</td>
<td>+10 x +2.6</td>
<td>+10 x -.0142</td>
<td>+10 x +.129</td>
</tr>
<tr>
<td>Long 10 November 90 puts</td>
<td>10 x -.23</td>
<td>+10 x -14</td>
<td>+10 x +2.2</td>
<td>+10 x -.0122</td>
<td>+10 x +.111</td>
</tr>
<tr>
<td>Short 20 November 90 calls</td>
<td>20 x +.25</td>
<td>-20 x +86</td>
<td>-20 x +2.2</td>
<td>-20 x -.0122</td>
<td>-20 x +.111</td>
</tr>
<tr>
<td>Long 20 November 95 puts</td>
<td>20 x -.19</td>
<td>+20 x +70</td>
<td>+20 x +3.5</td>
<td>+20 x +.0189</td>
<td>+20 x +.172</td>
</tr>
<tr>
<td>Long 10 November 90 puts</td>
<td>10 x -.23</td>
<td>+10 x -14</td>
<td>+10 x +2.2</td>
<td>+10 x -.0122</td>
<td>+10 x +.111</td>
</tr>
<tr>
<td>Short 20 November 95 puts</td>
<td>20 x +.22</td>
<td>-20 x +30</td>
<td>-20 x +3.5</td>
<td>-20 x -.0189</td>
<td>-20 x +.172</td>
</tr>
<tr>
<td>Long 10 November 100 puts</td>
<td>10 x -.07</td>
<td>+10 x -49</td>
<td>+10 x +4.0</td>
<td>+10 x -.0218</td>
<td>+10 x +.199</td>
</tr>
<tr>
<td>Long 10 November 110 calls</td>
<td>10 x +.07</td>
<td>+10 x -49</td>
<td>+10 x +4.0</td>
<td>+10 x -.0218</td>
<td>+10 x +.199</td>
</tr>
</tbody>
</table>
Sell 10 November 95 calls – 6.90 (theoretical value = 6.71)
Buy 7 futures contracts

<table>
<thead>
<tr>
<th>theoretical edge</th>
<th>delta</th>
<th>gamma</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+7 x 100</td>
<td>0</td>
</tr>
<tr>
<td>+10 x .19</td>
<td>-10 x 70</td>
<td>-10 x 3.5</td>
</tr>
<tr>
<td>+1.90</td>
<td>0</td>
<td>-35.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>theta</th>
<th>vega</th>
</tr>
</thead>
<tbody>
<tr>
<td>+7 x 0</td>
<td>+7 x 0</td>
</tr>
<tr>
<td>-10 x -.0387</td>
<td>-10 x .134</td>
</tr>
<tr>
<td>+.387</td>
<td>-1.34</td>
</tr>
</tbody>
</table>
Positive Delta: You want the underlying price to rise
Negative Delta: You want the underlying price to fall

Positive Gamma: You want the underlying contract to make a big move, or move very quickly
Negative Gamma: You want the underlying contract to sit still, or move very slowly

Positive Theta: The passage of time will help
Negative Theta: The passage of time will hurt

Positive Vega: You want implied volatility to rise
Negative Vega: You want implied volatility to fall
positive delta

negative gamma

(positive theta)

positive vega

slow upward movement

rising implied volatility
## Risk Interpretation Exercise

Match each position with the corresponding market conditions which will most help the position.

<table>
<thead>
<tr>
<th>position</th>
<th>market conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>+delta / +gamma / -vega</td>
<td>no price movement; rising implied volatility</td>
</tr>
<tr>
<td>-delta / -gamma / -vega</td>
<td>upward price movement; falling implied volatility</td>
</tr>
<tr>
<td>0 delta / -gamma / +vega</td>
<td>price movement in either direction; rising implied volatility</td>
</tr>
<tr>
<td>0 delta / +gamma / +vega</td>
<td>swift upward price movement; falling implied volatility</td>
</tr>
<tr>
<td>+delta / -gamma / +vega</td>
<td>downward price movement</td>
</tr>
<tr>
<td>0 delta / +gamma / -vega</td>
<td>price movement in either direction; falling implied volatility</td>
</tr>
<tr>
<td>-delta / 0 gamma / 0 vega</td>
<td>slow upward price movement; rising implied volatility</td>
</tr>
<tr>
<td>+delta / 0 gamma / -vega</td>
<td>slow downward price movement; falling implied volatility</td>
</tr>
</tbody>
</table>
An at-the-money option always has a greater gamma, theta, and vega than an equivalent in-the-money or out-of-the-money option.

A long-term option always has a greater vega than an equivalent short-term option.
**Theta values** (time decay)

As time passes the theta of an *at-the-money* option increases.

- 3 months to expiration: -0.03
- 3 weeks to expiration: -0.06
- 3 days to expiration: -0.16
Zhengzhou Commodity Exchange
Option Seminar

Dynamic Hedging
futures price = 99.75

time to November expiration = 10 weeks

volatility = 16.85%

November 100 call ??

theoretical value = 2.82

price = 2.60

How can we capture the difference between the option’s price and its theoretical value?
buy a call option

delta neutral or flat

take an opposing delta position in the underlying contract (-ΔC)
determine the option’s delta (ΔC)

current underlying price

theoretical value

underlying price
Due to the option’s *curvature*, as market conditions change the position will become unhedged.

ΔC

underlying price

unhedged amount

Current underlying price

theoretical value

ΔC

underlying price

unhedged amount

Current underlying price
Determine the new delta of the option.
Rehedge the position to return to delta neutral.
Continue the rehedging process throughout the life of the option.
Continue the rehedging process throughout the life of the option.  

**Dynamic Hedging**

\[ \Delta C \]
Each time the position becomes unhedged there is a potential profit opportunity. We can capture this profit by rehedging the position.

Suppose we add up all the profit opportunities over the life of the option which result from the rehedging process. What should this equal?

the option’s theoretical value

The rehedging process is a type of statistical arbitrage.
Delta of the November 100 call = 50 (.50)

For each November 100 call we buy, we must sell .50 of an underlying contract

Buy 100 November 100 calls

\[
\text{call delta} = +100 \times 50 = +5000
\]

Sell 50 futures contracts

\[
\text{futures delta} = -50 \times 100 = -5000
\]

Total position delta = +5000 -5000 = 0
**Delta Neutral:** Within a small range, no preference as to whether the underlying market moves up or down

Positive delta: Preference for upward movement

Negative delta: Preference for downward movement
One week later:

- futures price = 100.21 (previously 99.75)
- time to November expiration = 9 weeks
- volatility = 16.85%

Delta of November 100 call = 53

total delta position =

+100*53 -50*100 =

+5300 -5000 = +300
**Adjustment:** A trade made with the primary objective of returning the position to delta neutral

Previous delta position: $+300$

Adjustment: sell 3 futures contracts

New delta position: $+300 - 3\times 100 = 0$
Another week passes:

futures price  =  98.86 (previously 100.21)
time to November expiration  =  8 weeks
volatility  =  16.85%

Delta of November 100 call  =  44
total delta position  =

+100*44 -50*100 -300  =

+4400 -5000 -300  =  -900
Previous delta position:  -900

Adjustment:   buy 9 futures contracts

New delta position:  -900 +9*100 = 0

Dynamic Hedging:  the process of periodically adjusting a position in order to remain delta neutral

At expiration: close out the entire position at fair value
<table>
<thead>
<tr>
<th>week</th>
<th>futures price</th>
<th>delta of November 100 call</th>
<th>current delta position</th>
<th>adjustment</th>
<th>total adjustments</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>99.75</td>
<td>50</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>100.21</td>
<td>53</td>
<td>+300</td>
<td>sell 3</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>98.86</td>
<td>44</td>
<td>-900</td>
<td>buy 9</td>
<td>+6</td>
</tr>
<tr>
<td>3</td>
<td>97.12</td>
<td>33</td>
<td>-1100</td>
<td>buy 11</td>
<td>+17</td>
</tr>
<tr>
<td>4</td>
<td>98.72</td>
<td>42</td>
<td>+900</td>
<td>sell 9</td>
<td>+8</td>
</tr>
<tr>
<td>5</td>
<td>101.01</td>
<td>59</td>
<td>+1700</td>
<td>sell 17</td>
<td>-9</td>
</tr>
<tr>
<td>6</td>
<td>96.88</td>
<td>26</td>
<td>-3300</td>
<td>buy 33</td>
<td>+24</td>
</tr>
<tr>
<td>7</td>
<td>98.69</td>
<td>38</td>
<td>+1200</td>
<td>sell 12</td>
<td>+12</td>
</tr>
<tr>
<td>8</td>
<td>97.33</td>
<td>21</td>
<td>-1700</td>
<td>buy 17</td>
<td>+29</td>
</tr>
<tr>
<td>9</td>
<td>100.62</td>
<td>61</td>
<td>+4000</td>
<td>sell 40</td>
<td>-11</td>
</tr>
<tr>
<td>10</td>
<td>102.28</td>
<td></td>
<td></td>
<td>buy 11</td>
<td></td>
</tr>
</tbody>
</table>
**Original position**

Futures price at November expiration = 102.28

Value of the November 100 call at expiration

$$102.28 - 100 = 2.28$$

Option P&L = $100 \times (2.28 - 2.60)$

$$= 100 \times -0.32 = -32.00$$

Futures P&L = $50 \times (99.75 - 102.28)$

$$= 50 \times -2.53 = -126.50$$

Total P&L on original position

$$= -32.00 - 126.50 = -158.50$$
Adjustment process

week 1 – sell 3 futures contracts at 101.21
week 2 – buy 9 futures contracts at 98.86
week 3 – sell 11 futures contracts at 97.12
...
week 9 – sell 40 futures contracts at 100.62
week 10 – buy 11 futures contracts at 102.28

Total adjustment P&L: +180.57
P&L

Original position  
-158.50

Adjustment process  
+180.57

Total P&L  
+22.07

Predicted P&L:  
\[ 100 \times (2.82 - 2.60) = 100 \times 0.22 \]

= 22.00
Real-world considerations:

- transaction costs
- trading restrictions
- interest rates
- we don’t know the future volatility
Option trading based on theoretical evaluation can be thought of as a race between:

- the cash flow generated by the dynamic hedging process
- the decay in the option’s value as time passes

If the option is trading at a price greater than its value the decay wins the race.

If the option is trading at a price less than its value dynamic hedging wins the race.

If the option is trading at a price equal to its value the race is a tie.
Volatility = 16.85%

theoretical value = 2.82

If volatility turns out to be higher than 16.85% ……

the option is worth more than 2.82

If volatility turns out to be lower than 16.85% ……

the option is worth less than 2.82
As volatility declines the option’s theoretical value declines.

Price of the November 100 call = 2.60

At what volatility will the theoretical value of the November 100 call exactly equal its price?

At a volatility of 15.60%, the value of the November 100 call = 2.60

break-even volatility = 15.60%

implied volatility = 15.60%
**Delta Hedging Exercise**

For this question use the following table of delta values:

<table>
<thead>
<tr>
<th></th>
<th>June 70</th>
<th>June 75</th>
<th>June 80</th>
<th>June 85</th>
<th>June 90</th>
</tr>
</thead>
<tbody>
<tr>
<td>call delta</td>
<td>87</td>
<td>72</td>
<td>52</td>
<td>34</td>
<td>19</td>
</tr>
<tr>
<td>put delta</td>
<td>-13</td>
<td>-28</td>
<td>-48</td>
<td>-66</td>
<td>-81</td>
</tr>
</tbody>
</table>

You buy 25 June 80 calls. What do you need to do (buy? sell? how many?) to hedge your position as close to delta neutral as possible with each of the following contracts:

- underlying contract
  - June 85 call
  - June 75 put

You sell 80 June 75 puts. What do you need to do (buy? sell? how many?) to hedge your position as close to delta neutral as possible with each of the following contracts:

- June 70 call
- June 80 put
- June 90 call

You sell 15 underlying contracts. You would like to hedge half your delta position with June 70 puts and half your delta position with June 90 calls. As close as possible, how many of each contract do you need to buy or sell?
Introduction to Spreading
**Spread** – a position in one contract, or group of contracts, and an opposing position in a different contract, or group of contracts

Position?

- directional
- volatility
- interest rate
- delta
- gamma / vega
- rho
buy calls directional position?

positive delta

How might you spread off your directional risk?

sell underlying contracts

sell different calls

buy puts
buy puts

How might you spread off your gamma position?

sell different puts

sell calls
Why spread?

You believe there is a relative mispricing between contracts.

You want to construct a position which reflects a particular view of market conditions.

You need to manage the risk of your position.
Roulette bet

value = 95¢  price = $1.00

One player would like to bet $2,000 on one number. What can happen?

Player loses:  +$2,000

Player wins:  -$70,000
Roulette bet

\[ \text{value} = 95\,\text{¢} \quad \text{price} = \$1.00 \]

Two players would like to bet $1,000 each, but on different numbers.

Both players lose: \( +$2,000 \)

One player wins: \( -$34,000 \)
Roulette bet

\[
\text{value} = 95\,\text{¢} \quad \text{price} = \$1.00
\]

Situation 1: One player betting $2,000 on one number

Situation 2: Two players betting $1,000 each on different numbers

In the *long run* which situation is better for the casino?
Roulette bet

\[
\text{value} = 95\,\text{¢} \quad \text{price} = $1.00
\]

Situation 1: One player betting $2,000 on one number
\[
5\% \times $2,000 = $100
\]

Situation 2: Two players betting $1,000 each on different numbers
\[
5\% \times (2 \times $1,000) = $100
\]
Roulette bet

value = 95¢ price = $1.00

Situation 1: One player betting $2,000 on one number

Situation 2: Two players betting $1,000 each on different numbers

In the short run which situation is riskier for the casino?
Roulette bet

value = 95\(\text{¢}\) \hspace{1cm} price = $1.00

What is a perfect spread for the casino?
38 players betting $1,000 each on all 38 numbers

One player must win: -$36,000

Amount of money on the table: +$38,000

Casino wins $2,000
Spreading helps us ....

maintain the theoretical edge

while reducing the risk

Market-making:

get an edge  manage the risk
get an edge  manage the risk
get an edge  manage the risk
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Volatility Strategies
Volatility Spread

A spread, usually delta neutral, which is sensitive to either the volatility of the underlying contract (gamma), or to changes in implied volatility (vega)
Long Straddle

+1 September 100 call

+1 September 100 put

Short Straddle

-1 September 100 call

-1 September 100 put
**Long Straddle**

underlying price = 100.00

+1 September 100 call  
+50

+1 September 100 put  
-50

\[ \Delta = 0 \]
Long Straddle

underlying price = 105.00

+1 September 100 call +75
+3 September 100 put -25

Ratio Spread

Any spread where the number long market contracts and short market contracts are unequal
Long Straddle

exercise
price
Short Straddle

exercise price
Long Straddle

<table>
<thead>
<tr>
<th>delta</th>
<th>gamma</th>
<th>theta</th>
<th>vega</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Short Straddle

<table>
<thead>
<tr>
<th>delta</th>
<th>gamma</th>
<th>theta</th>
<th>vega</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>
Long Strangle

+1 September 95 put
+1 September 105 call

Short Strangle

-1 September 95 put
-1 September 105 call
**Long Strangle**

futures price = 100

+1 September 105 put \( \delta \) -75

+1 September 95 call \( \delta \) +75

0

**Guts**

A strangle where both options are in-the-money
Long Strangle

exercise price

exercise price
Short Strangle

exercise price

exercise price
### Long Strangle

<table>
<thead>
<tr>
<th>delta</th>
<th>gamma</th>
<th>theta</th>
<th>vega</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+</td>
<td>−</td>
<td>+</td>
</tr>
</tbody>
</table>

### Short Strangle

<table>
<thead>
<tr>
<th>delta</th>
<th>gamma</th>
<th>theta</th>
<th>vega</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>−</td>
<td>+</td>
<td>−</td>
</tr>
</tbody>
</table>
Long Butterfly

+1 September 95 call
-2 September 100 calls
+1 September 105 call

+1 November 90 put
-2 November 100 puts
+1 November 110 put
Short Butterfly

-1 September 95 call
+2 September 100 calls
-1 September 105 call
-1 November 90 put
+2 November 100 puts
-1 November 110 put
## Long Butterfly

<table>
<thead>
<tr>
<th></th>
<th>90</th>
<th>110</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1 September 95 call</td>
<td>0</td>
<td>+15</td>
<td>+5</td>
</tr>
<tr>
<td>-2 September 100 calls</td>
<td>0</td>
<td>-20</td>
<td>0</td>
</tr>
<tr>
<td>+1 September 105 call</td>
<td>0</td>
<td>+5</td>
<td>0</td>
</tr>
<tr>
<td>minimum value</td>
<td>0</td>
<td>0</td>
<td>+5</td>
</tr>
</tbody>
</table>

maximum value = amount between exercise prices
Long Butterfly

exercise price
exercise price
exercise price
Short Butterfly

exercise price  exercise price  exercise price
Long Butterfly

<table>
<thead>
<tr>
<th>delta</th>
<th>gamma</th>
<th>theta</th>
<th>vega</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Short Butterfly

<table>
<thead>
<tr>
<th>delta</th>
<th>gamma</th>
<th>theta</th>
<th>vega</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>
long straddle
+gamma / +vega

long strangle
+gamma / +vega

short butterfly
+gamma / +vega

short straddle
-gamma / -vega

short strangle
-gamma / -vega

long butterfly
-gamma / -vega
**Ratio Spread** (buy more than sell)

futures price = 100

<table>
<thead>
<tr>
<th></th>
<th>delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>+3 September 105 call</td>
<td>25</td>
</tr>
<tr>
<td>-1 September 95 call</td>
<td>75</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>+2 November 95 put</td>
<td>-25</td>
</tr>
<tr>
<td>-1 November 100 put</td>
<td>-50</td>
</tr>
</tbody>
</table>

0
**Ratio Spread** (sell more than buy)

<table>
<thead>
<tr>
<th>Futures Price</th>
<th>Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>-3</td>
</tr>
<tr>
<td>-3 September 105 call</td>
<td>25</td>
</tr>
<tr>
<td>+1 September 95 call</td>
<td>75</td>
</tr>
<tr>
<td>-2 November 95 put</td>
<td>-25</td>
</tr>
<tr>
<td>+1 November 100 put</td>
<td>-50</td>
</tr>
</tbody>
</table>

Delta:
-3 + 1 = 0

Delta:
-2 + 1 = 0
futures price = 100

<table>
<thead>
<tr>
<th>Call</th>
<th>Price</th>
<th>Price Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>+3 September 105 call</td>
<td>1.00</td>
<td>+3 * 14.00 = +23.00</td>
</tr>
<tr>
<td>-1 September 95 call</td>
<td>6.00</td>
<td>-3 * 1.00 = -2.00</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{120} & \quad +3 \times 14.00 & \quad -1 \times 19.00 & \quad = \quad +23.00 \\
\text{80} & \quad -3 \times 1.00 & \quad +1 \times 6.00 & \quad = \quad +3.00 \\
\text{100} & \quad -3 \times 1.00 & \quad +1 \times 1.00 & \quad = \quad -2.00 \\
\end{align*}
\]
Call Ratio Spread (buy more than sell)

sell the lower exercise price  buy the higher exercise price
Call Ratio Spread (sell more than buy)

- buy the lower exercise price
- sell the higher exercise price
Put Ratio Spread (buy more than sell)

buy the lower exercise price

sell the higher exercise price
Put Ratio Spread (sell more than buy)

sell the lower exercise price

buy the higher exercise price
**Ratio Spread** – buy more than sell

\[
\begin{array}{ccccc}
\text{delta} & \text{gamma} & \text{theta} & \text{vega} \\
0 & + & - & + \\
\end{array}
\]

**Ratio Spread** – sell more than buy

\[
\begin{array}{ccccc}
\text{delta} & \text{gamma} & \text{theta} & \text{vega} \\
0 & - & + & - \\
\end{array}
\]
<table>
<thead>
<tr>
<th></th>
<th>downside risk / reward</th>
<th>upside risk / reward</th>
</tr>
</thead>
<tbody>
<tr>
<td>long straddle</td>
<td>unlimited reward</td>
<td>unlimited risk</td>
</tr>
<tr>
<td>short straddle</td>
<td>unlimited risk</td>
<td>unlimited reward</td>
</tr>
<tr>
<td>long butterfly</td>
<td>limited risk</td>
<td>limited risk</td>
</tr>
<tr>
<td>short butterfly</td>
<td>limited reward</td>
<td>limited reward</td>
</tr>
<tr>
<td></td>
<td>downside risk / reward</td>
<td>upside risk / reward</td>
</tr>
<tr>
<td>------------------</td>
<td>-------------------------</td>
<td>----------------------</td>
</tr>
<tr>
<td><strong>call ratio spread</strong></td>
<td>limited reward</td>
<td>unlimited reward</td>
</tr>
<tr>
<td>(buy more than sell)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>put ratio spread</strong></td>
<td>unlimited reward</td>
<td>limited reward</td>
</tr>
<tr>
<td>(buy more than sell)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>call ratio spread</strong></td>
<td>limited risk</td>
<td>unlimited risk</td>
</tr>
<tr>
<td>(sell more than buy)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>put ratio spread</strong></td>
<td>unlimited risk</td>
<td>limited risk</td>
</tr>
<tr>
<td>(sell more than buy)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Long Calendar Spread
(Time Spread, Horizontal Spread)

+1 November 100 call
-1 September 100 call

+1 May 95 put
-1 January 95 put
Short Calendar Spread
(Time Spread, Horizontal Spread)

-1 November 100 call
+1 September 100 call

-1 May 95 put
+1 January 95 put
+1 November 100 call
-1 September 100 call

<table>
<thead>
<tr>
<th>Month</th>
<th>4 months</th>
<th>3 months</th>
<th>2 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>November</td>
<td>3.00</td>
<td>2.60</td>
<td>2.10</td>
</tr>
<tr>
<td>September</td>
<td>2.10</td>
<td>1.30</td>
<td>0</td>
</tr>
</tbody>
</table>

futures price = 100

<table>
<thead>
<tr>
<th>Month</th>
<th>4 months</th>
<th>3 months</th>
<th>2 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>November</td>
<td>.90</td>
<td>1.30</td>
<td>2.10</td>
</tr>
<tr>
<td>September</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
+1 November 100 call
-1 September 100 call

<table>
<thead>
<tr>
<th></th>
<th>100</th>
<th>150</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>November</td>
<td>3.00</td>
<td>50.05</td>
<td>.05</td>
</tr>
<tr>
<td>September</td>
<td>2.10</td>
<td>50.00</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>.90</td>
<td>.05</td>
<td>.05</td>
</tr>
</tbody>
</table>
+1 November 100 call
-1 September 100 call

<table>
<thead>
<tr>
<th></th>
<th>25%</th>
<th>30%</th>
<th>20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>November</td>
<td>3.00</td>
<td>3.90</td>
<td>2.10</td>
</tr>
<tr>
<td>September</td>
<td>2.10</td>
<td>2.50</td>
<td>1.70</td>
</tr>
</tbody>
</table>

---

.90  1.40  .40

*negative gamma / positive vega*
Long Calendar Spread

exercise price

value at near-term expiration
Short Calendar Spread

exercise price

value at near-term expiration
Long Calendar Spread

<table>
<thead>
<tr>
<th>delta</th>
<th>gamma</th>
<th>theta</th>
<th>vega</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>–</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Short Calendar Spread

<table>
<thead>
<tr>
<th>delta</th>
<th>gamma</th>
<th>theta</th>
<th>vega</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
Volatility Spreads

<table>
<thead>
<tr>
<th>delta</th>
<th>gamma</th>
<th>theta</th>
<th>vega</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>
Volatility Spreads

gamma / vega

+ + long straddle, long strangle, short butterfly, ratio spread (buy more than sell)

− − short straddle, short strangle, long butterfly, ratio spread (sell more than buy)

− + long calendar spread

+ − short calendar spread
Volatility Strategy Exercise

On this and the following pages are several different volatility strategies with some possible changes in market conditions. If the underlying futures contract is currently trading at 80, for each change in market conditions is the strategy making money (+) or losing money (−). Assume all positions are initially delta neutral.

<table>
<thead>
<tr>
<th>Scenario Description</th>
<th>Position 1</th>
<th>Position 2</th>
<th>Position 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>the underlying price rises sharply</td>
<td>+1 June 80 call</td>
<td>+1 June 80 put</td>
<td>implied volatility falls</td>
</tr>
<tr>
<td>time passes with no change in the underlying volatility</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>the underlying price falls sharply</td>
<td>+2 August 75 puts</td>
<td>-1 August 85 put</td>
<td>implied volatility rises</td>
</tr>
<tr>
<td>time passes with no change in the underlying volatility</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>the underlying price falls sharply</td>
<td>-1 March 80 call</td>
<td>+1 January 80 call</td>
<td>implied volatility rises</td>
</tr>
<tr>
<td>time passes with no change in the underlying volatility</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Volatility Strategy Exercise

- The underlying price falls sharply
  -1 September 75 put
  +2 September 80 puts
  -1 September 80 put

- The underlying price rises sharply
  +3 July 85 calls
  -1 July 75 call

- The underlying price falls sharply
  -4 October 70 puts
  +1 October 85 put

- The underlying price rises sharply
  +1 December 80 put
  -1 September 80 put

- Time passes with no change in the underlying
- Implied volatility rises
  
- Time passes with no change in the underlying
- Implied volatility falls
  
- Time passes with no change in the underlying
- Implied volatility rises
  
- Time passes with no change in the underlying
- Implied volatility falls
  
196
### Volatility Strategy Exercise

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Action</th>
<th>Time Passes with No Change</th>
<th>Implied Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>The underlying price</td>
<td>+1 August 75 call</td>
<td></td>
<td>Falls</td>
</tr>
<tr>
<td>falls sharply</td>
<td>-2 August 85 calls</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The underlying price</td>
<td>-1 May 85 call</td>
<td></td>
<td>Rises</td>
</tr>
<tr>
<td>rises sharply</td>
<td>-1 May 75 put</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The underlying price</td>
<td>+1 November 80 call</td>
<td></td>
<td>Rises</td>
</tr>
<tr>
<td>rises sharply</td>
<td>-1 October 80 call</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The underlying price</td>
<td>-1 April 80 call</td>
<td></td>
<td>Rises</td>
</tr>
<tr>
<td>rises sharply</td>
<td>-1 April 80 put</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Zhengzhou Commodity Exchange
Option Seminar

Bull & Bear Strategies
Futures price = 100

Long Straddle

+1 December 100 call
+1 December 100 put
delta neutral

Bull Straddle

+1 December 95 call
+1 December 95 put
+1 December 100 call
-1 December 110 call

bull spread

-1 December 100 call
+1 December 110 call

bear spread

minimum value = 0

maximum value = $X_h - X_l$
+1 December 100 put  
-1 December 110 put  \textit{bull spread}

-1 December 100 put  
+1 December 110 put  \textit{bear spread}

minimum value = 0

maximum value = X_h - X_l
**Bull (Vertical) Spread**

Buy an option at a lower exercise price

Sell an option at a higher exercise price

**Bear (Vertical) Spread**

Buy an option at a higher exercise price

Sell an option at a lower exercise price

Both options must be the same type (both calls or both puts) and expire at the same time.
Bull (Vertical) Spread

- buy the lower exercise price
- sell the higher exercise price
Bear (Vertical) Spread

sell the lower exercise price

buy the higher exercise price
Futures price = 100
Time to expiration = 6 weeks

<table>
<thead>
<tr>
<th>Volatility</th>
<th>20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>95 call</td>
<td>5.86 (78)</td>
</tr>
<tr>
<td>95 / 100 spread</td>
<td>3.15 (27)</td>
</tr>
<tr>
<td>100 call</td>
<td>2.71 (51)</td>
</tr>
<tr>
<td>100 / 105 spread</td>
<td>1.75 (27)</td>
</tr>
<tr>
<td>105 call</td>
<td>.96 (24)</td>
</tr>
</tbody>
</table>
Futures price = 100
Time to expiration = 6 weeks

<table>
<thead>
<tr>
<th>Volatility</th>
<th>20%</th>
<th>15%</th>
<th>25%</th>
</tr>
</thead>
<tbody>
<tr>
<td>95 call</td>
<td>5.86</td>
<td>5.41</td>
<td>6.38</td>
</tr>
<tr>
<td>95 / 100 spread</td>
<td>3.15</td>
<td>3.38</td>
<td>3.00</td>
</tr>
<tr>
<td>100 call</td>
<td>2.71</td>
<td>2.03</td>
<td>3.38</td>
</tr>
<tr>
<td>100 / 105 spread</td>
<td>1.75</td>
<td>1.56</td>
<td>1.86</td>
</tr>
<tr>
<td>105 call</td>
<td>.96</td>
<td>.47</td>
<td>1.52</td>
</tr>
</tbody>
</table>
Futures price = 100
Time to expiration = 6 weeks

<table>
<thead>
<tr>
<th>Volatility</th>
<th>20%</th>
<th>110</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>95 call</td>
<td>5.86</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>95 / 100 spread</td>
<td>3.15</td>
<td>+</td>
<td>-</td>
<td>○+</td>
</tr>
<tr>
<td>100 call</td>
<td>2.71</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100 / 105 spread</td>
<td>1.75</td>
<td>○+</td>
<td>○-</td>
<td>-</td>
</tr>
<tr>
<td>105 call</td>
<td>.96</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Futures price = 100  
Time to expiration = 6 weeks

<table>
<thead>
<tr>
<th>Volatility</th>
<th>20%</th>
<th>15%</th>
<th>25%</th>
</tr>
</thead>
<tbody>
<tr>
<td>95 put</td>
<td>.86</td>
<td>.41</td>
<td>1.38</td>
</tr>
<tr>
<td>95 / 100 spread</td>
<td>1.85</td>
<td>1.62</td>
<td>2.00</td>
</tr>
<tr>
<td>100 put</td>
<td>2.71</td>
<td>2.03</td>
<td>3.38</td>
</tr>
<tr>
<td>100 / 105 spread</td>
<td>3.25</td>
<td>3.44</td>
<td>3.14</td>
</tr>
<tr>
<td>105 put</td>
<td>5.96</td>
<td>5.47</td>
<td>6.52</td>
</tr>
</tbody>
</table>
For each spread on this and the following page you are given an outlook for the underlying futures market (either bullish or bearish) and an opinion about implied volatility (you believe it is either unreasonably high or unreasonably low). Given this information, from the four suggested spreads choose the spread which you think is best.

**underlying price** | **directional outlook** | **implied volatility**
--- | --- | ---
long 50 call / short 45 call |  |  
long 45 call / short 50 call |  |  
long 40 put / short 35 put | 50 | bearish  
long 55 put / short 50 put |  | too high

long 70 call / short 75 call |  |  
long 70 put / short 75 put | 71 | bearish  
long 70 call / short 65 call |  | too low
long 65 call / short 70 call
<table>
<thead>
<tr>
<th>underlying price</th>
<th>directional outlook</th>
<th>implied volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>long 40 put / short 45 put</td>
<td>39</td>
<td>bullish</td>
</tr>
<tr>
<td>long 35 put / short 40 put</td>
<td></td>
<td></td>
</tr>
<tr>
<td>long 45 put / short 40 put</td>
<td>82</td>
<td>bullish</td>
</tr>
<tr>
<td>long 45 call / short 4 call</td>
<td></td>
<td></td>
</tr>
<tr>
<td>long 80 call / short 75 call</td>
<td></td>
<td></td>
</tr>
<tr>
<td>long 80 put / short 75 put</td>
<td></td>
<td></td>
</tr>
<tr>
<td>long 75 call / short 80 call</td>
<td>1700</td>
<td>bearish</td>
</tr>
<tr>
<td>long 85 call / short 90 call</td>
<td></td>
<td></td>
</tr>
<tr>
<td>long 1900 put / short 1600 put</td>
<td></td>
<td></td>
</tr>
<tr>
<td>long 1800 call / short 1700 call</td>
<td>1700</td>
<td>bearish</td>
</tr>
<tr>
<td>long 1700 call / short 1500 call</td>
<td></td>
<td></td>
</tr>
<tr>
<td>long 1800 put / short 2000 put</td>
<td></td>
<td></td>
</tr>
<tr>
<td>long 80 call / short 75 call</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Zhengzhou Commodity Exchange
Option Seminar

Synthetics
+1 December 100 call
-1 December 100 put

above 100
put is worthless / exercise call
buy underlying at 100

below 100
call is worthless / assigned on put
buy underlying at 100
Regardless of whether the underlying is above or below the exercise price at expiration

-1 December 100 put

+1 December 100 call

buy underlying at 100

**Synthetic Long Underlying**

*(Synthetic Long Forward)*
long call
short put

exercise
price

long underlying
long call + short put ≈

**synthetic long underlying**

short call + long put ≈

**synthetic short underlying**

delta of an underlying contract? 100

+1 December 100 call
-1 December 100 put

+30 +80
-70 -20
long call + short put ≈

**synthetic long underlying**

short call + long put ≈

**synthetic short underlying**

gamma of an underlying contract? 0

+1 December 100 call  5  2
-1 December 100 put  5  2
long call + short put ≈ 

*synthetic long underlying*

short call + long put ≈ 

*synthetic short underlying*

vega of an underlying contract? 0

+1 December 100 call .15 .40

-1 December 100 put .15 .40
long put + long underlying ≈ synthetic long call

short put + short underlying ≈ synthetic short call

long call + short underlying ≈ synthetic long put

short call + long underlying ≈ synthetic short put
Buy December 100 straddle:

+1 December 100 call
+1 December 100 put
+2 December 100 calls
-1 futures contract
+2 December 100 puts
+1 futures contract
Bull spread:

buy a lower exercise price
sell a higher exercise price

+1 December 90 call
-1 December 100 call

+1 December 90 put
-1 December 100 put
Synthetic Equivalent Exercise

What is the strategy or synthetic equivalent for each combination below?

- long a Jan 75 put / long a futures contract
- short a Feb 85 call / long a futures contract
- long a Mar 65 call / long a Mar 65 put
- short a Apr 70 call / long a Apr 70 put
- long a May 75 put / short a May 85 put
- long a Jun 85 call / short a futures contract
- short a Jul 70 call / short a Jul 70 put
- long an Aug 90 call / short an Aug 90 put
- long a Sep 70 call / short a Sep 75 call
- short an Oct 65 put / short a futures contract
- short a Nov 90 call / short a Nov 65 put
-1 December 100 call
+1 December 100 put
+1 underlying contract

Arbitrage – buying and selling the same or closely related products in different markets in order to take advantage of a price discrepancy
Conversion

long underlying + synthetic short underlying

long underlying + short call
long put

The call and put have the same exercise price and expiration date.
Reverse Conversion (Reversal)

short underlying + synthetic long underlying

short underlying + long call
short put

The call and put have the same exercise price and expiration date.
Conversion

**buy** the underlying / **buy** the put

Reverse Conversion

**sell** the underlying / **sell** the put
Conversion

-1 December 100 call  6.00
+1 December 100 put  2.00
+1 futures contract  104.00

Cash flow:  +6   -2   +100  =  104

C - P = S - X  

*Put-Call Parity*
Conversion

buy  -1 December 100 call  6.00
sell  +1 December 100 put  2.00
sell  +1 futures contract

Cash flow:  +6  -2  +100  =  104

C - P  =  S - X  
6 - 2  =  104.50 - 100

4  ≠  4.50
When fixing settlement prices, exchanges try to maintain put-call parity.
Synthetic Pricing Exercise - Futures

For each set of futures options below, fill in the missing value

<table>
<thead>
<tr>
<th>futures price</th>
<th>exercise price</th>
<th>call price</th>
<th>put price</th>
</tr>
</thead>
<tbody>
<tr>
<td>19560</td>
<td>19000</td>
<td>325</td>
<td></td>
</tr>
<tr>
<td>2803</td>
<td>1375</td>
<td>205</td>
<td>252</td>
</tr>
<tr>
<td>9838</td>
<td>9800</td>
<td>125</td>
<td>62</td>
</tr>
<tr>
<td>2319</td>
<td>8600</td>
<td>133</td>
<td>133</td>
</tr>
<tr>
<td>5225</td>
<td>311</td>
<td>236</td>
<td></td>
</tr>
<tr>
<td>3890</td>
<td>3800</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
Zhengzhou Commodity Exchange
Option Seminar

Hedging Strategies
Options as “insurance”

Practical considerations

How much protection do I need?

How much risk am I willing to accept?

Theoretical considerations

What is the cost of the insurance?

Does the premium I receive fairly compensate me for the lost opportunity?
Options as “insurance”

Buy a protective option

long an underlying contract – buy a put
short an underlying contract – buy a call

Advantage – absolute, well-defined protection; unlimited potential profit

Disadvantage – cost of option; loss of premium
long an underlying contract
long a protective put

exercise price

option premium

long call
short an underlying contract
long a protective call

exercise price

long put

option premium
Options as insurance

Sell a covered option

long an underlying contract — sell a call
short an underlying contract — sell a put

Advantage — receive premium

Disadvantage — sold option offers only partial protection; limited profit potential
long an underlying contract
short a covered call

\[ \text{exercise price} \]

\{ \text{option premium} \}

\textit{short put}
short an underlying contract
short a covered put
Options as insurance

Simultaneously buy a protective option and sell a covered option

long an underlying contract
  – buy a put / sell a call (long collar)

short an underlying contract
  – buy a call / sell a put (short collar)

Zero-cost collar – the price of the bought and sold options are the same
long an underlying contract
long put
short call

bull spread

exercise price
exercise price
short an underlying contract
long call
short put

exercise price
exercise price

bear spread
Hedging strategies tend to reduce the volatility of a portfolio.

Is that desirable?

<table>
<thead>
<tr>
<th>Year 1 Returns</th>
<th>Year 2 Returns</th>
<th>Year 3 Returns</th>
<th>Average Return</th>
<th>Total Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>+25%</td>
<td>-20%</td>
<td>+25%</td>
<td>+10%</td>
<td>+25%</td>
</tr>
<tr>
<td>+29%</td>
<td>-34%</td>
<td>+44%</td>
<td>+13%</td>
<td>+22.6%</td>
</tr>
<tr>
<td>+16%</td>
<td>-6%</td>
<td>+17%</td>
<td>+9%</td>
<td>+27.6%</td>
</tr>
<tr>
<td>+8.5%</td>
<td>+8.5%</td>
<td>+8.5%</td>
<td>+8.5%</td>
<td>+27.7%</td>
</tr>
</tbody>
</table>
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Answers to Exercises
Expected Value Exercise (answers)

<table>
<thead>
<tr>
<th>Price</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>124</td>
<td>4%</td>
</tr>
<tr>
<td>126</td>
<td>12%</td>
</tr>
<tr>
<td>128</td>
<td>19%</td>
</tr>
<tr>
<td>130</td>
<td>22%</td>
</tr>
<tr>
<td>132</td>
<td>21%</td>
</tr>
<tr>
<td>134</td>
<td>16%</td>
</tr>
<tr>
<td>136</td>
<td>6%</td>
</tr>
</tbody>
</table>

Using the above prices and probabilities for an underlying contract, what are the expected values for the following contracts:

<table>
<thead>
<tr>
<th>Contract</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>130.32</td>
<td>4.40</td>
</tr>
<tr>
<td>130</td>
<td>1.42</td>
</tr>
<tr>
<td>126 put</td>
<td>0.08</td>
</tr>
<tr>
<td>130 put</td>
<td>1.10</td>
</tr>
<tr>
<td>133 put</td>
<td>3.02</td>
</tr>
</tbody>
</table>

What do you notice about the difference between the values of calls and puts at the same exercise price? **They differ by intrinsic value.**
**Volatility Exercise I** (answers)

For each contract and volatility below, what would be an approximate daily and weekly standard deviation:

<table>
<thead>
<tr>
<th></th>
<th>10%</th>
<th>12%</th>
<th>14%</th>
<th>16%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sugar futures trading at 5140</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>daily</td>
<td>32</td>
<td>39</td>
<td>45</td>
<td>51</td>
</tr>
<tr>
<td>weekly</td>
<td>71</td>
<td>86</td>
<td>100</td>
<td>114</td>
</tr>
<tr>
<td><strong>Glass futures trading at 1465</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>daily</td>
<td>14</td>
<td>18</td>
<td>23</td>
<td>27</td>
</tr>
<tr>
<td>weekly</td>
<td>31</td>
<td>41</td>
<td>51</td>
<td>61</td>
</tr>
</tbody>
</table>
Volatility Exercise II (answers)

For each contract, volatility, and time interval below, what would be an approximate one standard deviation price change:

Cotton futures trading at 19,600

- volatility = 7.5%, time = 22 days
  \[ 19600 \times 0.075 \times \sqrt{\frac{22}{365}} = 361 \]
- volatility = 11.25%, time = 86 days
  \[ 19600 \times 0.1125 \times \sqrt{\frac{86}{365}} = 1070 \]

Wheat futures trading at 2625.00

- volatility = 14%, time = 9 weeks
  \[ 2625 \times 0.14 \times \sqrt{\frac{9}{52}} = 153 \]
- volatility = 9.75%, time = 27 weeks
  \[ 2625 \times 0.0975 \times \sqrt{\frac{27}{52}} = 184 \]
**Risk Measurement Exercise** (answers)

<table>
<thead>
<tr>
<th></th>
<th>new theoretical value using a constant delta</th>
<th>new average delta</th>
<th>new theoretical value using the average delta</th>
<th>if ten days pass</th>
<th>if volatility changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>original delta</td>
<td>new theoretical value using a constant delta</td>
<td>new average delta</td>
<td>new theoretical value using the average delta</td>
<td>if ten days pass</td>
<td>if volatility changes</td>
</tr>
<tr>
<td>a) 65</td>
<td>9.99</td>
<td>76.1</td>
<td>70.6</td>
<td>10.16</td>
<td>7.68</td>
</tr>
<tr>
<td>b) -28</td>
<td>1.18</td>
<td>-22.3</td>
<td>-25.1</td>
<td>1.25</td>
<td>1.67</td>
</tr>
<tr>
<td>c) 50</td>
<td>3.04</td>
<td>42.9</td>
<td>46.5</td>
<td>3.09</td>
<td>3.64</td>
</tr>
<tr>
<td>d) -87</td>
<td>19.45</td>
<td>-94.8</td>
<td>-90.9</td>
<td>19.56</td>
<td>16.52</td>
</tr>
<tr>
<td>e) 11</td>
<td>1.02</td>
<td>12.3</td>
<td>11.6</td>
<td>1.03</td>
<td>.93</td>
</tr>
<tr>
<td>f) -44</td>
<td>18.96</td>
<td>-100</td>
<td>-72.0</td>
<td>21.76</td>
<td>14.11</td>
</tr>
</tbody>
</table>
**Risk Interpretation Exercise** (answers)

Match each position with the corresponding market conditions which will most help the position.

<table>
<thead>
<tr>
<th>position</th>
<th>market conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>+delta / +gamma / -vega</td>
<td>swift upward price movement; falling implied volatility</td>
</tr>
<tr>
<td>-delta / -gamma / -vega</td>
<td>slow downward price movement; falling implied volatility</td>
</tr>
<tr>
<td>0 delta / -gamma / +vega</td>
<td>no price movement; rising implied volatility</td>
</tr>
<tr>
<td>0 delta / +gamma / +vega</td>
<td>price movement in either direction; rising implied volatility</td>
</tr>
<tr>
<td>+delta / -gamma / +vega</td>
<td>slow upward price movement; rising implied volatility</td>
</tr>
<tr>
<td>0 delta / +gamma / -vega</td>
<td>price movement in either direction; falling implied volatility</td>
</tr>
<tr>
<td>-delta / 0 gamma / 0 vega</td>
<td>downward price movement</td>
</tr>
<tr>
<td>+delta / 0 gamma / -vega</td>
<td>upward price movement; falling implied volatility</td>
</tr>
</tbody>
</table>
**Delta Hedging Exercise** (answers)

For this question use the following table of delta values:

<table>
<thead>
<tr>
<th></th>
<th>June 70</th>
<th>June 75</th>
<th>June 80</th>
<th>June 85</th>
<th>June 90</th>
</tr>
</thead>
<tbody>
<tr>
<td>call delta</td>
<td>87</td>
<td>72</td>
<td>52</td>
<td>34</td>
<td>19</td>
</tr>
<tr>
<td>put delta</td>
<td>-13</td>
<td>-28</td>
<td>-48</td>
<td>-66</td>
<td>-81</td>
</tr>
</tbody>
</table>

You buy 25 June 80 calls. What do you need to do (buy? sell? how many?) to hedge your position as close to delta neutral as possible with each of the following contracts:

- **underlying contract:**
  - June 85 call: sell 13
  - June 75 put: buy 46

You sell 80 June 75 puts. What do you need to do (buy? sell? how many?) to hedge your position as close to delta neutral as possible with each of the following contracts:

- **June 70 call:** sell 26
- **June 80 put:** buy 47

You sell 15 underlying contracts. You would like to hedge half your delta position with June 70 puts and half your delta position with June 90 calls. As close as possible, how many of each contract do you need to buy or sell?

- **sell 58 June 70 puts**
- **buy 39 June 90 calls**
Volatility Strategy Exercise (answers)

On this and the following pages are several different volatility strategies with some possible changes in market conditions. If the underlying futures contract is currently trading at 80, for each change in market conditions is the strategy making money (+) or losing money (−). Assume all positions are initially delta neutral.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>The underlying price rises sharply</td>
<td>+</td>
</tr>
<tr>
<td>+1 June 80 call</td>
<td></td>
</tr>
<tr>
<td>+1 June 80 put</td>
<td></td>
</tr>
<tr>
<td>Time passes with no change in the underlying implied volatility falls</td>
<td>−</td>
</tr>
<tr>
<td>The underlying price falls sharply</td>
<td>+</td>
</tr>
<tr>
<td>+2 August 75 puts</td>
<td></td>
</tr>
<tr>
<td>-1 August 85 put</td>
<td></td>
</tr>
<tr>
<td>Time passes with no change in the underlying implied volatility rises</td>
<td>+</td>
</tr>
<tr>
<td>-1 March 80 call</td>
<td></td>
</tr>
<tr>
<td>+1 January 80 call</td>
<td></td>
</tr>
</tbody>
</table>

249
Volatility Strategy Exercise (answers)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Trade Position</th>
<th>Time Passes</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>The underlying price falls sharply</td>
<td>-1 September 75 put +3 July 85 calls +1 December 80 put -4 October 70 puts +1 October 85 put</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>The underlying price rises sharply</td>
<td>+2 September 80 puts +1 July 75 call</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>The underlying price rises sharply</td>
<td>-1 September 85 put</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>
Volatility Strategy Exercise (answers)

<table>
<thead>
<tr>
<th>Situation</th>
<th>Underlying Price</th>
<th>Time Change</th>
<th>Implied Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1 August 75 call, -2 August 85 calls</td>
<td>price falls sharply</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>-1 May 85 call, -1 May 75 put</td>
<td>price rises sharply</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>+1 November 80 call, -1 October 80 call</td>
<td>price rises sharply</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>-1 April 80 call, -1 April 80 put</td>
<td>price rises sharply</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>
Bull and Bear Strategy Exercise (answers)

For each spread on this and the following slide you are given an outlook for the underlying market (either bullish or bearish) and an opinion about implied volatility (you believe it is either unreasonably high or unreasonably low). Given this information, from the four suggested spreads choose the spread which you think is best.

<table>
<thead>
<tr>
<th>Spread</th>
<th>Underlying Price</th>
<th>Directional Outlook</th>
<th>Implied Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>long 50 call / short 45 call</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>long 45 call / short 50 call</td>
<td>50</td>
<td>bearish</td>
<td>too high</td>
</tr>
<tr>
<td>long 40 put / short 35 put</td>
<td>50</td>
<td>bearish</td>
<td>too high</td>
</tr>
<tr>
<td>long 55 put / short 50 put</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>long 70 call / short 75 call</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>long 70 put / short 75 put</td>
<td>71</td>
<td>bearish</td>
<td>too low</td>
</tr>
<tr>
<td>long 70 call / short 65 call</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>long 65 call / short 70 call</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Bull and Bear Strategy Exercise** (answers)

<table>
<thead>
<tr>
<th>underlying price</th>
<th>directional outlook</th>
<th>implied volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>long 40 put / short 45 put</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>long 35 put / short 40 put</td>
<td>39</td>
<td>bullish</td>
</tr>
<tr>
<td>long 45 put / short 40 put</td>
<td></td>
<td></td>
</tr>
<tr>
<td>long 45 call / short 4 call</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| **long 75 call / short 80 call**  |
| long 80 call / short 75 call |
| long 80 put / short 75 put |
| **long 75 call / short 80 call**  |
| long 85 call / short 90 call |

| **long 75 call / short 80 call**  |
| long 85 call / short 90 call |

| **long 1800 call / short 1700 call**  |
| long 1900 put / short 1600 put |
| long 1800 call / short 1700 call |
| **long 1800 call / short 1700 call**  |
| long 1700 call / short 1500 call |
| long 1800 put / short 2000 put | 1700 | bearish | too high |
Synthetic Equivalent Exercise (answers)

What is the strategy or synthetic equivalent for each combination below?

- long a Jan 75 put / long a futures contract
  - long a Jan 75 call

- short a Feb 85 call / long a futures contract
  - short a Feb 85 put

- long a Mar 65 call / long a Mar 65 put
  - long straddle

- short a Apr 70 call / long a Apr 70 put
  - short futures contract

- long a May 75 put / short a May 85 put
  - bull put spread

- long a Jun 85 call / short a futures contract
  - long a Jun 85 put

- short a Jul 70 call / short a Jul 70 put
  - short straddle

- long an Aug 90 call / short an Aug 90 put
  - long futures contract

- long a Sep 70 call / short a Sep 75 call
  - bull call spread

- short an Oct 65 put / short a futures contract
  - short an Oct 65 call

- short a Nov 90 call / short a Nov 65 put
  - short strangle
**Synthetic Pricing Exercise - Futures** (answers)

For each set of futures options below, fill in the missing value

<table>
<thead>
<tr>
<th>futures price</th>
<th>exercise price</th>
<th>call price</th>
<th>put price</th>
</tr>
</thead>
<tbody>
<tr>
<td>19560</td>
<td>19000</td>
<td>325</td>
<td>235</td>
</tr>
<tr>
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<td><strong>2850</strong></td>
<td>205</td>
<td>252</td>
</tr>
<tr>
<td><strong>1438</strong></td>
<td>1375</td>
<td>125</td>
<td>62</td>
</tr>
<tr>
<td>9838</td>
<td>9800</td>
<td><strong>65</strong></td>
<td>27</td>
</tr>
<tr>
<td><strong>8600</strong></td>
<td>8600</td>
<td>133</td>
<td>133</td>
</tr>
<tr>
<td>2319</td>
<td>2350</td>
<td>97</td>
<td><strong>166</strong></td>
</tr>
<tr>
<td>5225</td>
<td><strong>5150</strong></td>
<td>311</td>
<td>236</td>
</tr>
<tr>
<td>3890</td>
<td>3800</td>
<td><strong>90</strong></td>
<td>0</td>
</tr>
</tbody>
</table>